Alignment and regularized reconstructions in tomography as optimization problems

IX Conferencia de Matemáticos Ecuatorianos en París

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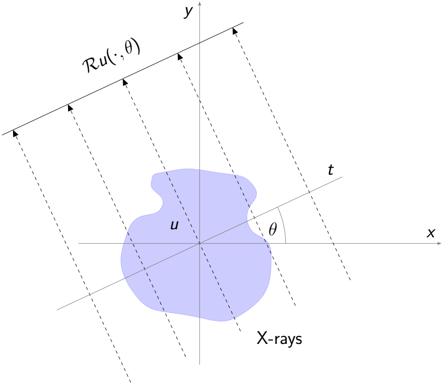
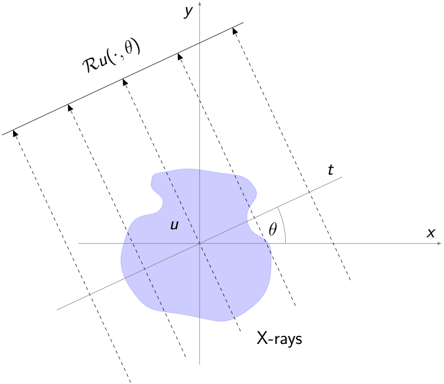
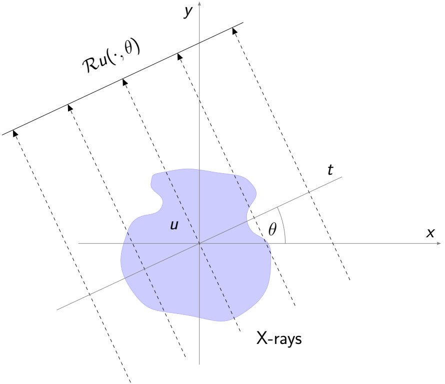


- u: object to be measured
- $v := \mathcal{R}u$: Radon transform of u: X-ray projections or sinogram

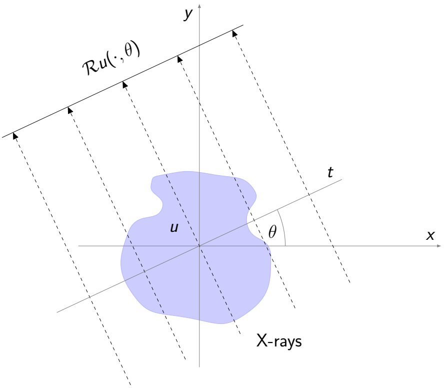


- Tomographic inverse problem:

$$\text{given } v \in \mathfrak{R}(\mathcal{R}), \text{ find } u \in U \text{ such that } \mathcal{R}u = v$$



- What we measure: misaligned sinogram $\tilde{v}(t, \theta) = v(t-h, \theta)$, $h \in \mathbb{R}$



- What we measure: misaligned sinogram $\tilde{v}(t,\theta) = v(t-h,\theta)$, $h \in \mathbb{R}$
- Tomographic *alignment* inverse problem

$$\text{given } \tilde{v}, \text{ find } h \in \mathbb{R} \text{ such that } \tilde{v}(\cdot + h, \cdot) \in \mathfrak{R}(\mathcal{R})$$

Introduction, alignment in parallel tomography
- example

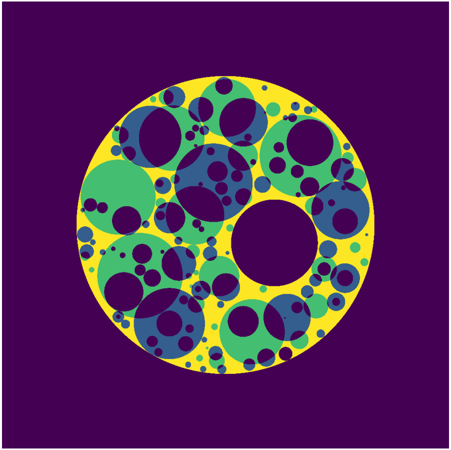
Fan-beam tomography
- alignment via fixed point iteration

Cone-beam tomography
- alignment via variable projection
- on data
- on real data

After alignment, regularized reconstruction

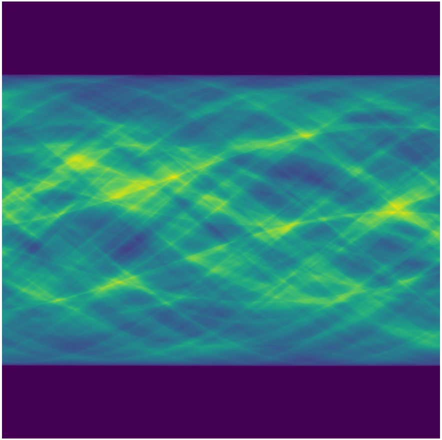
Conclusions and perspectives

- Numerical phantom[1]

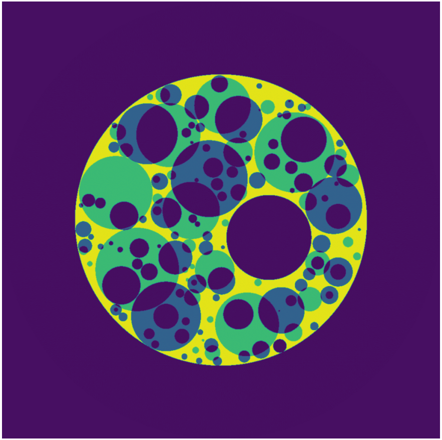


[1] based on the `foam_ct_phantom` library

- Conventional (FBP) reconstruction[2]



$\theta \in [0, \pi[$
$\mathcal{R}f$ (sinogram)

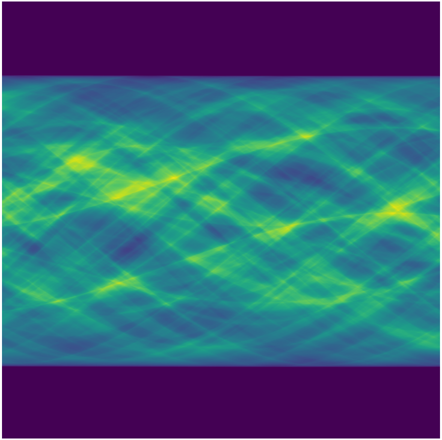


FBP reconstruction

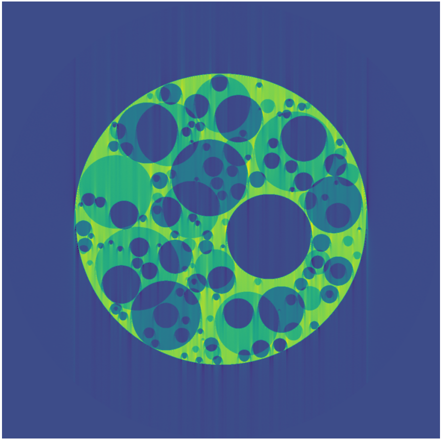
[2] Kak, Slaney. 1988

- Conventional (FBP) reconstruction

t



$\theta \in [0, \pi[$
$\mathcal{R}f$, $h = 2$ pixels



FBP reconstruction

- Parenthesis: alignment for parallel projections, based on

$$v(t,\theta) = v(-t,\theta+\pi), \quad \forall\,(t,\theta)$$

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- With $q = t + h$

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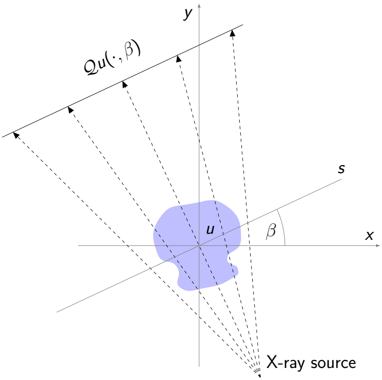
- Then, h is found with 1D signal registration:

$$h^* = \frac{1}{2}\,\text{shift}(\tilde{v}(\cdot,0),\tilde{v}(-\cdot,\pi)) = \frac{1}{2}\operatorname*{argmax}_{t\in\mathbb{R}}\,\tilde{v}(\cdot,0)\star\tilde{v}(-\cdot,\pi)$$



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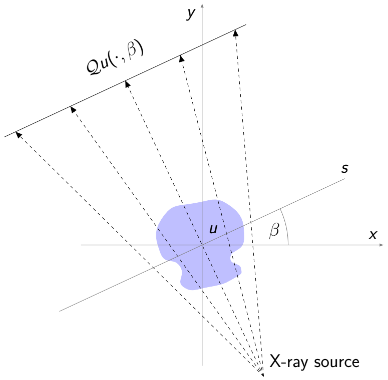

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- What we measure: $\tilde{v}(s,\beta) = v(s-h,\beta)$, $h \in \mathbb{R}$

- Fan-beam symmetry:

$$v(s,\beta) = v(-s, \beta + 2\arctan\frac{s}{r} + \pi), \quad \forall\,(s,\beta)$$

r : source-object distance, or source radius

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$$\tilde{v}(q, 0) = \tilde{v}(-q + 2h, 2 \arctan \frac{q - h}{r} + \pi), \quad \forall q \qquad (1)$$

- Denote the operators

$$\begin{cases} \Lambda v(q) = v(q, 0) \\ \Pi_h v(q) = v(-q + 2h, 2\arctan\frac{q-h}{r} + \pi) \end{cases}$$

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- and find h^* by solving

$$\min_h \{ L(h) := \|\Lambda \tilde{g} - \Pi_h \tilde{g}\|_2^2 \}$$

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- Works on real fan-beam data: < 20 iterations

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Lemma (arxiv:2310.09567, 2023)

$\tilde{v}$ verifies

$$\Lambda \tilde{v}(q) \approx \Pi_0 \tilde{v}(q - 2h^*), \quad \forall q \in \mathbb{R}$$

with an error bounded by

$$\max_q \left| \Lambda \tilde{v}(q) - \Pi_0 \tilde{v}(q - 2h^*) \right| \leq C_{h^*},$$

with

$$C_{h^*} = \max_{q,\beta} \left| \tilde{v}(q, \beta) - \tilde{v}(q, \beta + \frac{2h^*}{r}) \right|$$

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- From (1), we have

$$\mathrm{shift}(\Lambda\tilde{v}, \Pi_{h^*}\tilde{v}) = 0$$

thus h^* is a solution of

$$T_{\tilde{v}}(h) = h$$

i.e., h^* is a fixed point of $T_{\tilde{v}}$

Theorem (arxiv:2310.09567, 2023)

If the error C_{h^*} is such that $T_{\tilde{v}}$ is a contraction in a neighbourhood of h^*. The fanbeam alignment problem has a unique solution as the limit of the iteration

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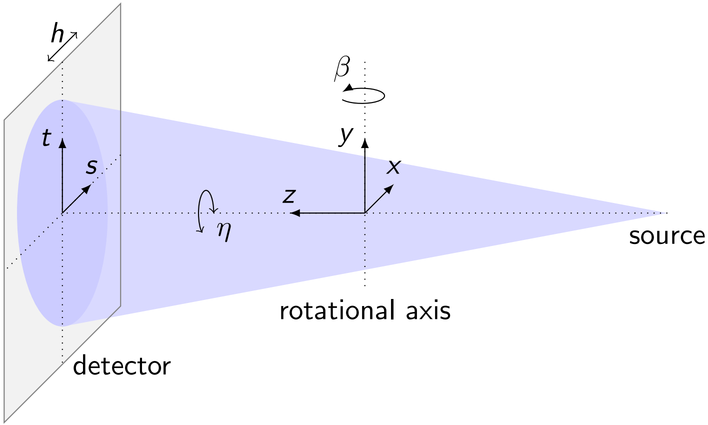
- 2D interpolations required only in

$$\Pi_{h_k}\tilde{v} = \tilde{v}(-q + 2h_k, 2\arctan\frac{q - h_k}{r} + \pi)$$

- Works on real fan-beam data: <5 iterations



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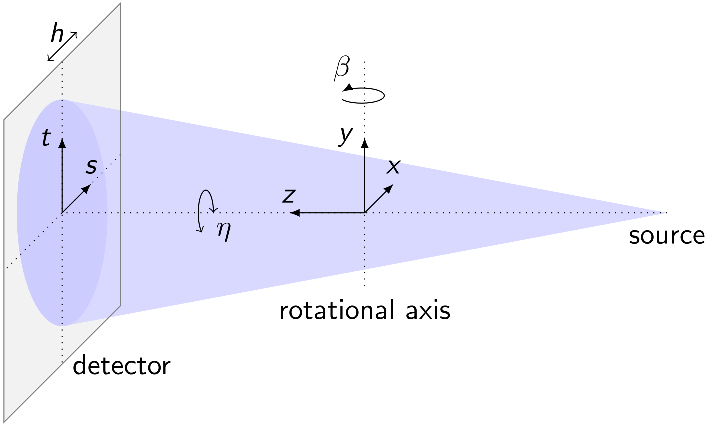

- $v := \mathcal{C}u$: Cone-beam transform of u parameterized with (s, t, β)
- Alignment variables to estimate: $\{h, \eta\}$

- Notations: Translation and rotation operators τ_h, κ_η :

$$\begin{cases} \tau_h v(s,t,\beta) = v(s-h,v,\beta) \\ \kappa_\eta v(u,v,\beta) = v(s\cos\eta - t\sin\eta, s\sin\eta + t\cos\eta, \beta) \end{cases}$$

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- We measure now :

$$\tilde{v}(s,t,\beta) = \tau_h \kappa_\eta v(s,t,\beta)$$

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- It is clear that $\kappa_\eta^{-1}\tau_h^{-1}\tilde{v} = \kappa_{-\eta}\tau_{-h}\tilde{v} = v$ then we have

$$\kappa_{-\eta}\tau_{-h}\tilde{v}(s,0,\beta) = \kappa_{-\eta}\tau_{-h}\tilde{v}(-s,0,\beta + 2\arctan\frac{s}{r} + \pi) \qquad (2)$$

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$$\begin{cases} \Lambda_{h,\eta} v(s,\beta) := \kappa_{-\eta}\tau_{-h} v(s,0,\beta) \\ \Pi_{h,\eta} v(s,\beta) := \kappa_{-\eta}\tau_{-h} v(-s,0,\beta + 2\arctan\frac{s}{r} + \pi) \end{cases}$$

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- then (h^*, η^*) can be estimated by solving :

$$\min_{h,\eta} \{ L(h,\eta) \coloneqq \| \Lambda_{h,\eta}\tilde{v} - \Pi_{h,\eta}\tilde{v} \|_2^2 \}$$

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- Too slow with standard quasi-newton methods

- Variable projection (VP) approach[3]

[3] Golub, Pereyra. SIAM J. Num. Analysis. 1973

- Variable projection (VP) approach[3]
- Project h onto η :

$$h^*(\eta) = \underset{h}{\operatorname{argmin}}\, L(h, \eta)$$

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with a *tilted* fanbeam fixed point approach **FP** previously introduced

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$$h^*(\eta) = \underset{h}{\operatorname{argmin}}\, L(h,\eta) = \mathbf{FP}(\eta)$$

with a *tilted* fanbeam fixed point approach **FP** previously introduced

- and finally solve

$$\eta^* = \underset{\eta}{\operatorname{argmin}}\{\bar{L}(\eta) := \|\Lambda_{h^*(\eta),\eta}\tilde{v} - \Pi_{h^*(\eta),\eta}\tilde{v}\|_2^2\}$$

[3] Golub, Pereyra. SIAM J. Num. Analysis. 1973

Theorem (Aravkin, van Leeuwen. 2012)

If L twice-differentiable and locally convex then:

$$\frac{d}{d\eta}\bar{L}(\eta) = \nabla_\eta L(h^*(\eta), \eta)$$

and a local minimum η^* *of* $\bar{L}$ *with* $h^*(\eta^*)$ *is a local minimum of L*

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- The algorithm[4]:

$$\begin{cases} h_k = \mathbf{FP}(\eta_k) \\ \eta_{k+1} = \eta_k - \gamma_k \nabla_\eta L(h_k, \eta_k) \end{cases}$$

[4] arxiv:2310.09567, 2023

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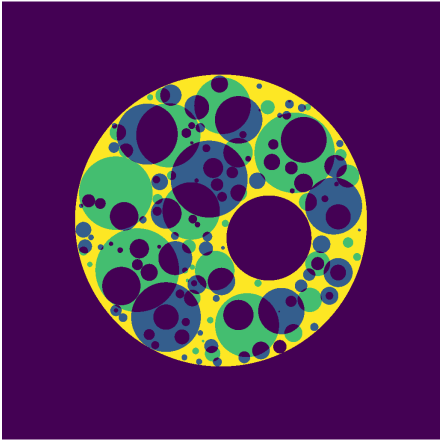
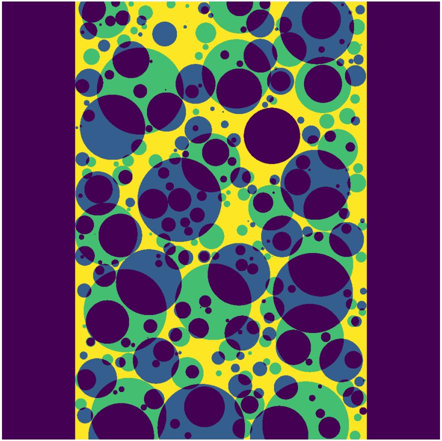
- Depends on data, but works in < 5 minutes for 2000^3 CT data with CPU python code and the Armijo step size rule[5] for γ_k

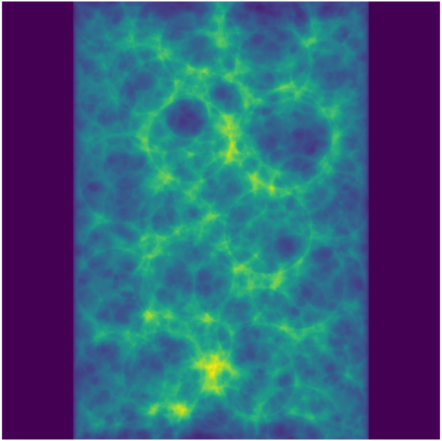
[4] arxiv:2310.09567, 2023
[5] Nocedal, Wright. 2006



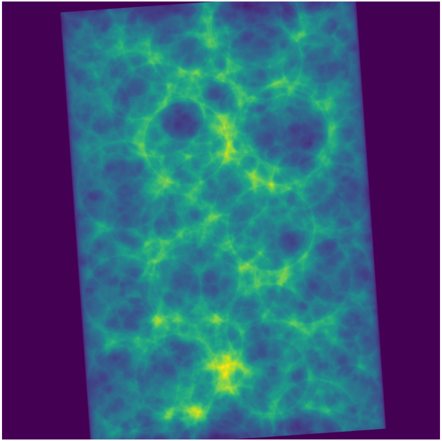
slice ($y = 0$)



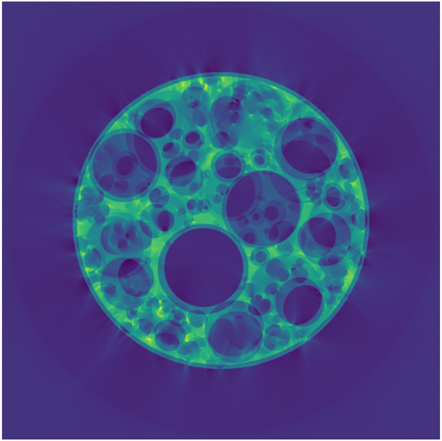
parallel to detector ($z = 0$)



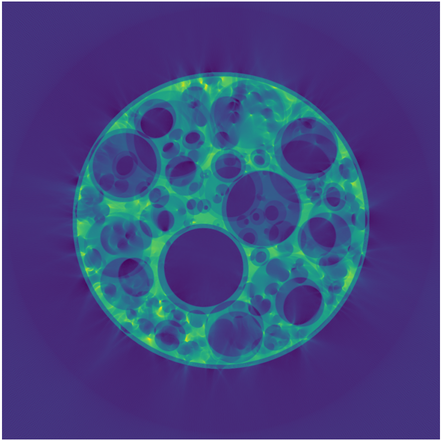
ideal conebeam projection



misaligned, $h = 5$ pix, $\eta = 4^\circ$



raw middle slice



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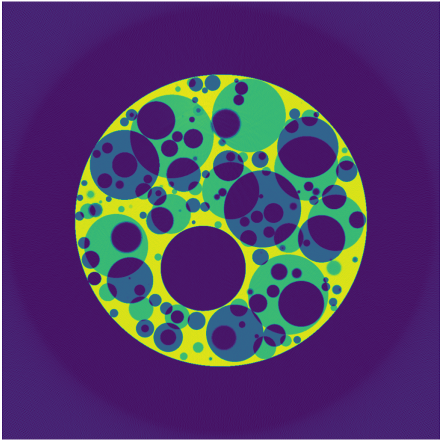
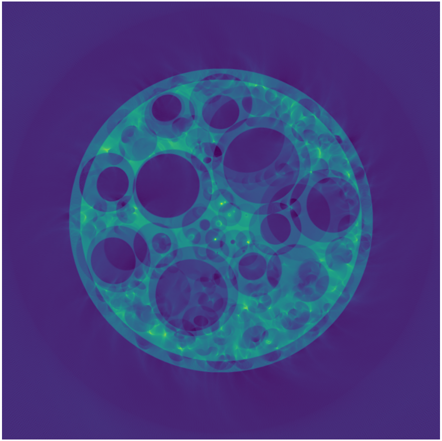
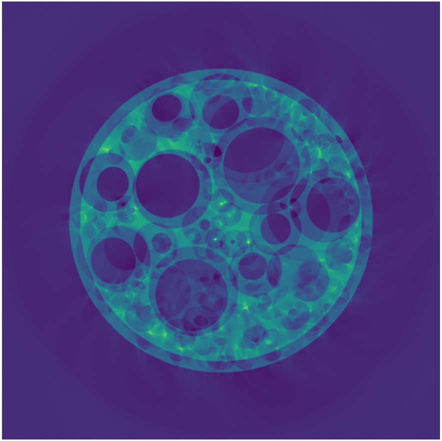


VP aligned middle slice



raw $\frac{1}{4}$ slice



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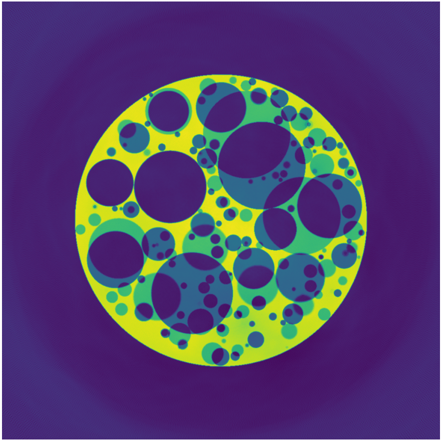
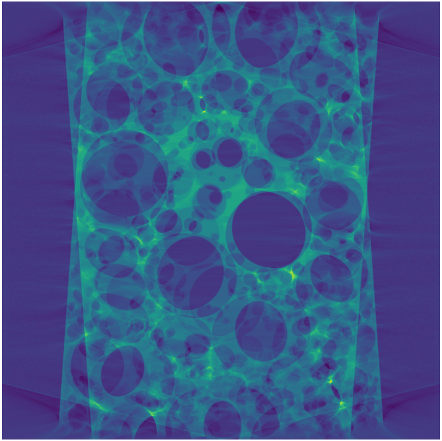
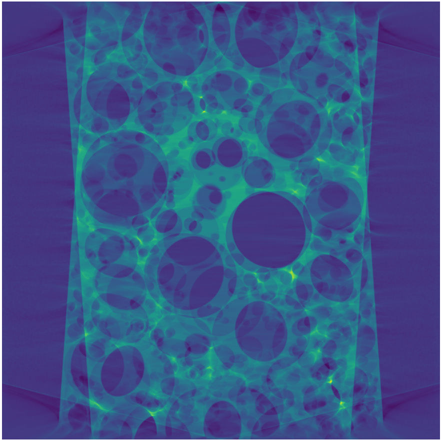


VP aligned $\frac{1}{4}$ slice



raw $z = 0$ slice



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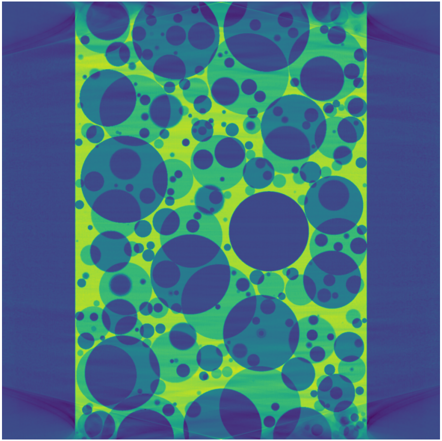
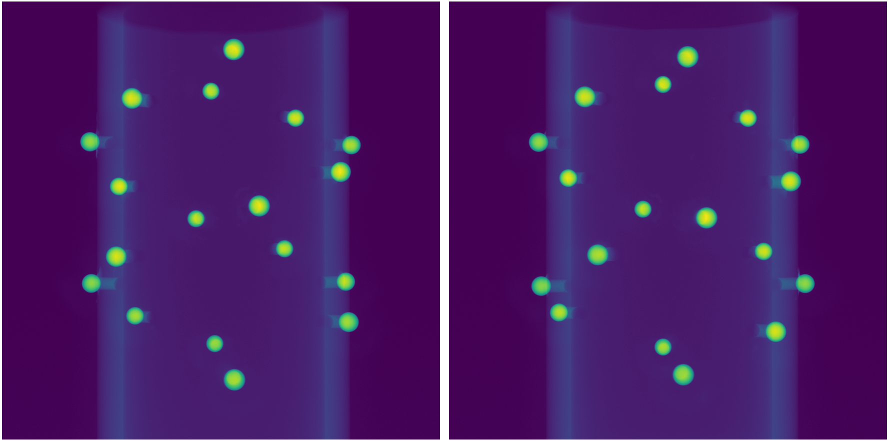


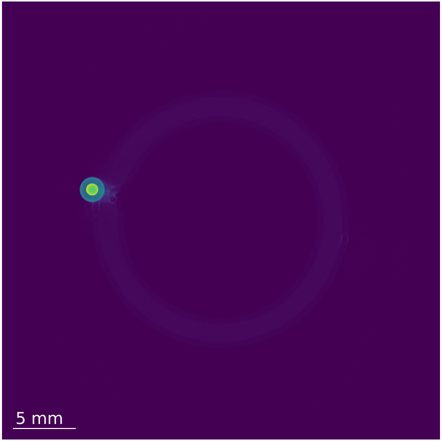
VP aligned $z = 0$ slice

- Calibration object[6]

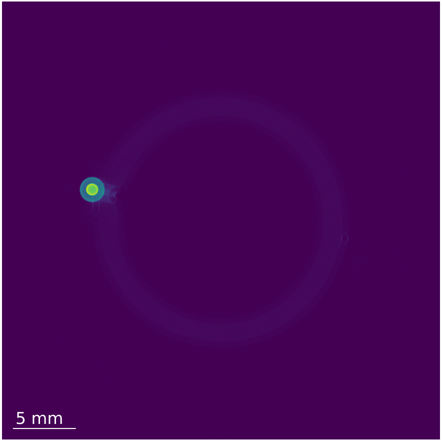


$\beta = 0$ $\beta = \pi$

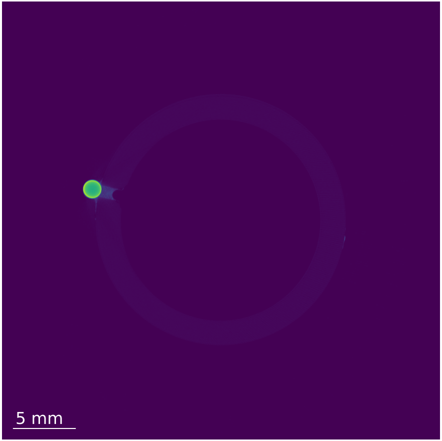
[6] M. Ferucci et al, Prec. Eng. 2021



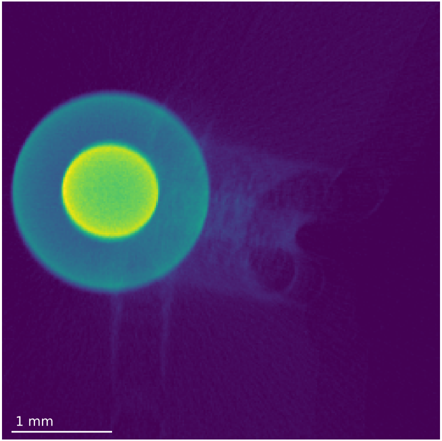
raw middle slice



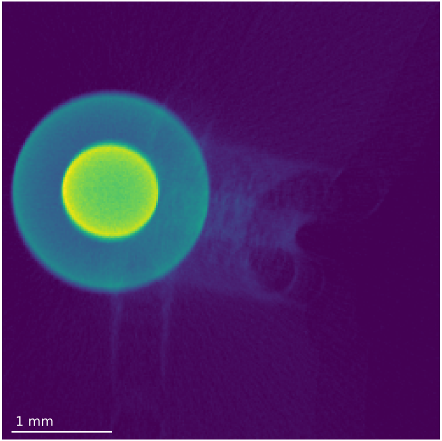
raw middle slice



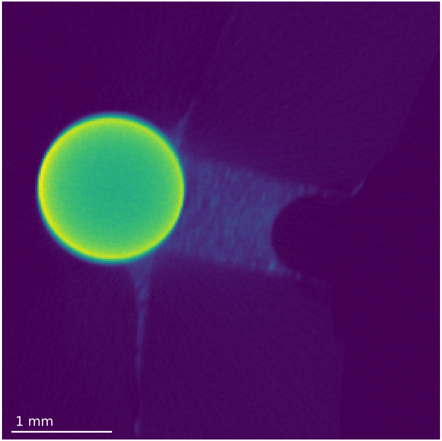
VP aligned



raw middle slice



raw middle slice



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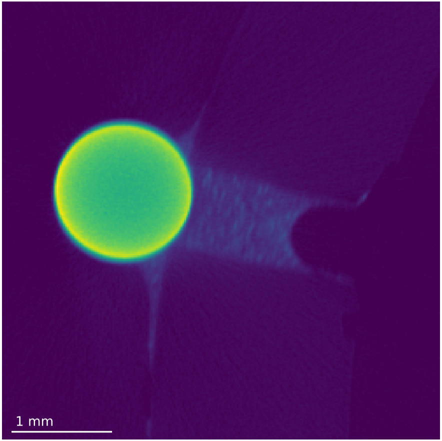
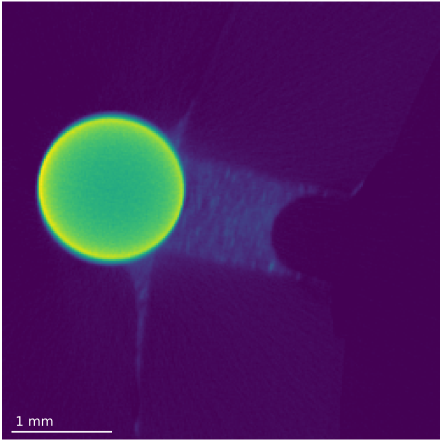


Reference-based method



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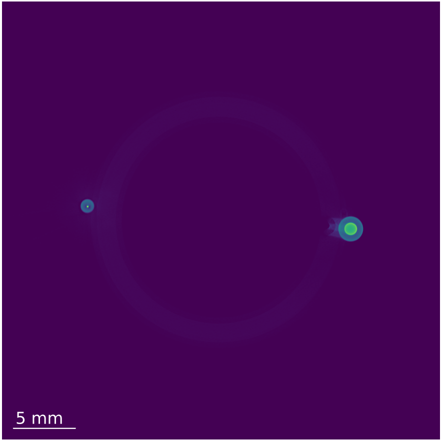
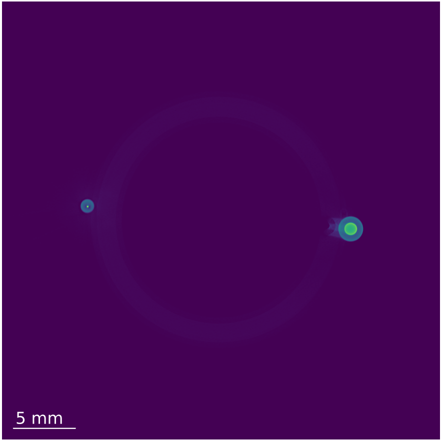
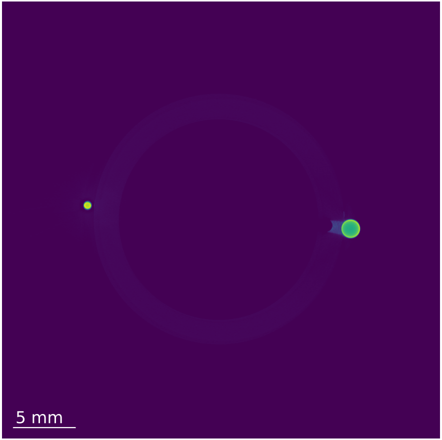


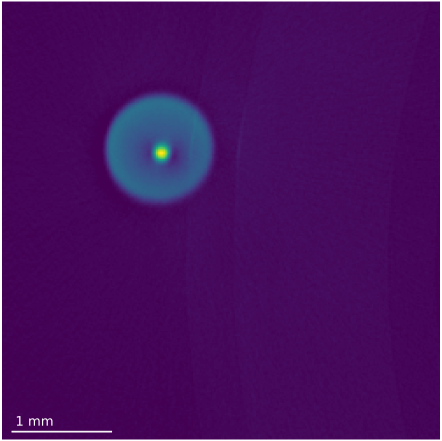
raw $\frac{1}{4}$ slice



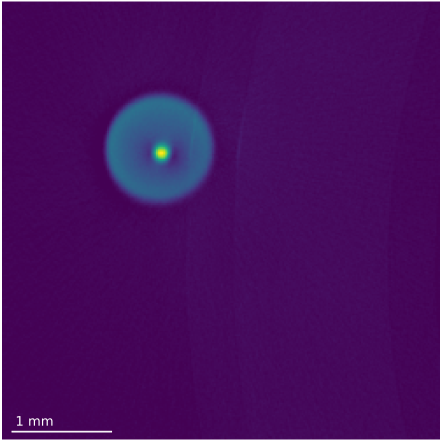
raw $\frac{1}{4}$ slice



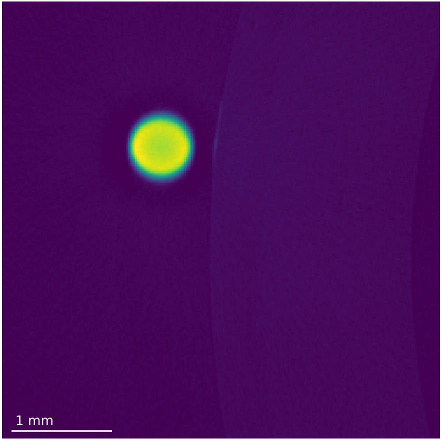
VP aligned



raw $\frac{1}{4}$ slice

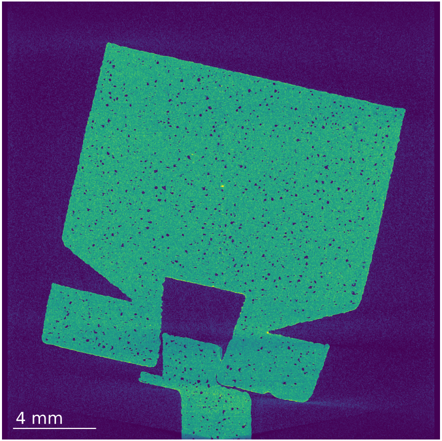


raw $\frac{1}{4}$ slice

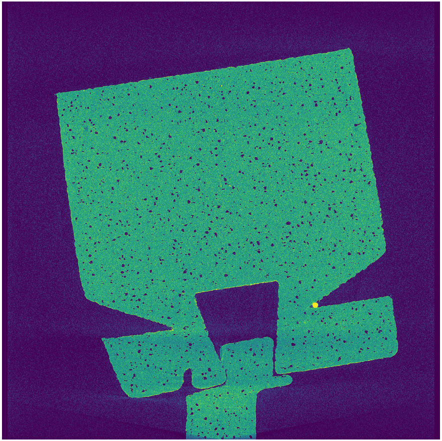


VP aligned

- Additive manufactured sample[7]



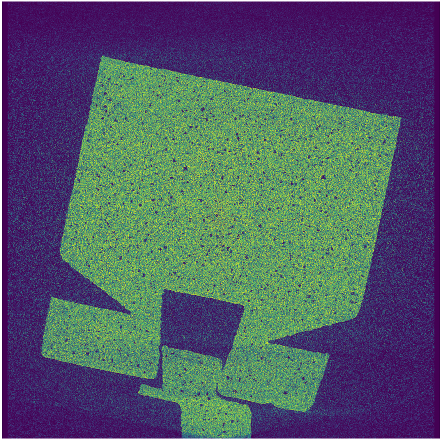
$z = 0$, VP aligned



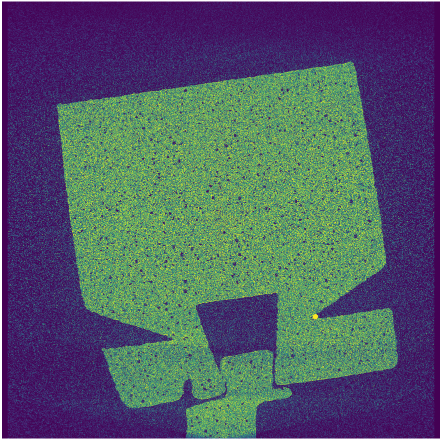
$y = 0$, VP aligned

[7] printed by R. Santander, KU Leuven

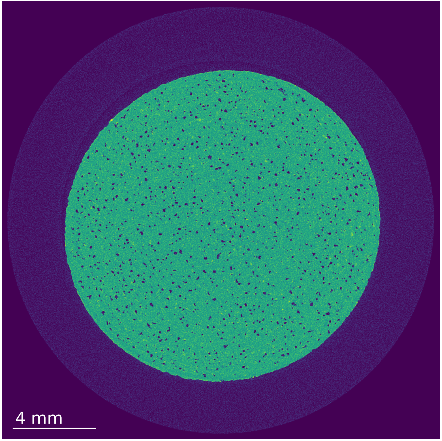
- Low quality (LQ) 5 min CT scan - high noise level



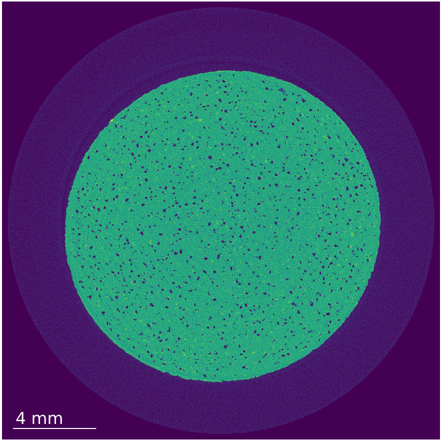
$z = 0$, VP aligned

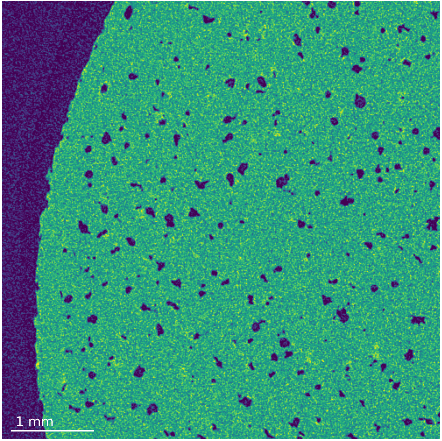


$y = 0$, VP aligned

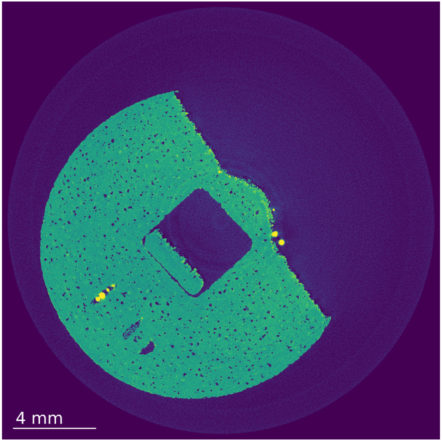


middle slice

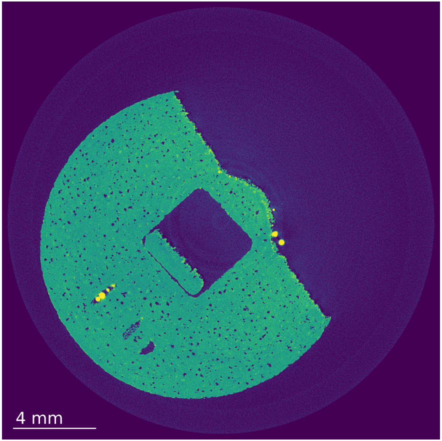




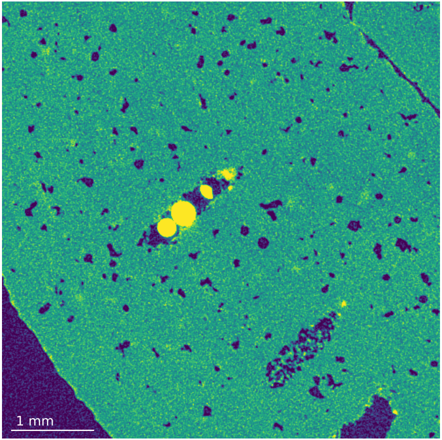
middle slice

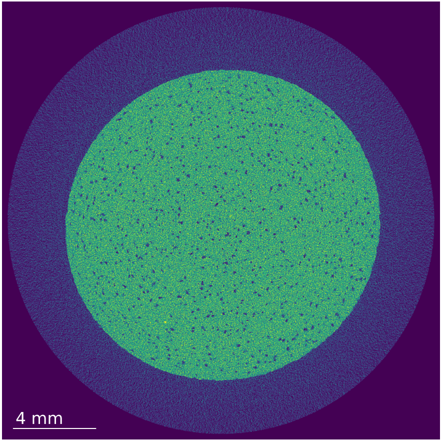


$\frac{1}{4}$ slice

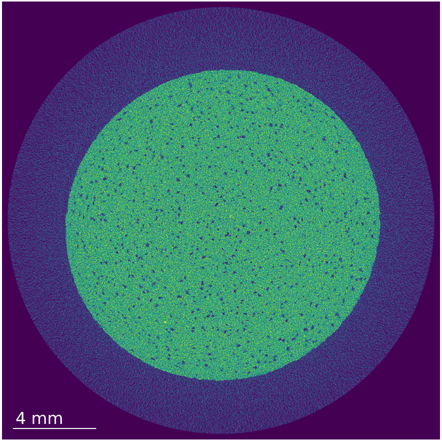


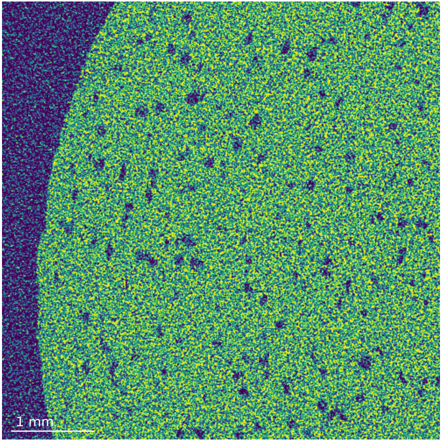
$\frac{1}{4}$ slice



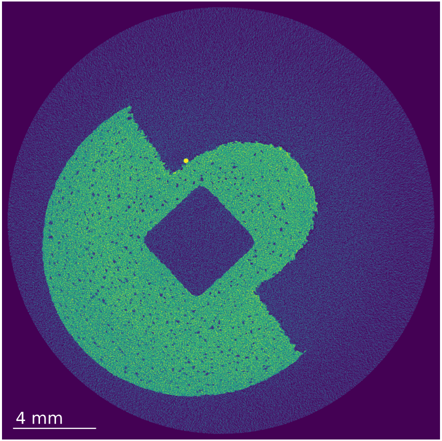


middle slice, LQ scan

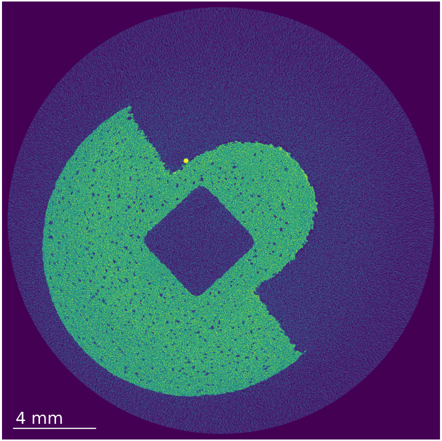


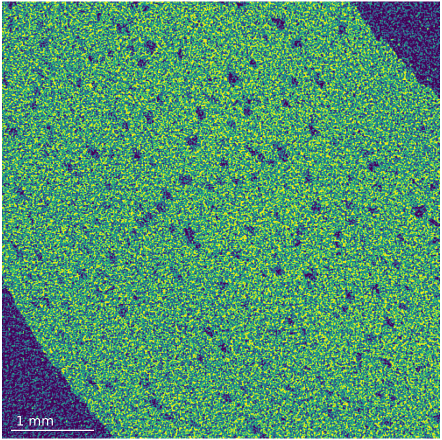


middle slice, LQ scan



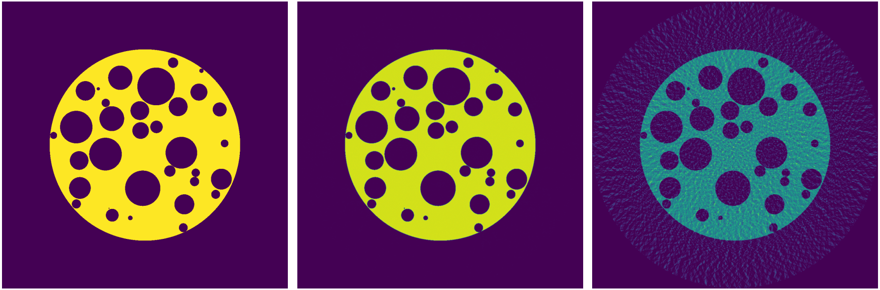
$\frac{1}{4}$ slice, LQ scan





$\frac{1}{4}$ slice, LQ scan

- Motivation: undersampled data



Phantom FBP 1024 projections FBP 60 projections

- Instead of FBP, we pose the problem as

$$\min_{u \in U} f(u) + g(Lu), \quad f, g \text{ convex}, \quad f \text{ smooth}, \quad L \text{ linear} \qquad \text{(PP)}$$

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- Recall the conjugate f^* of f

$$f^*(w) = \sup_{u\in U} \langle w, u\rangle - f(u)$$

then we have the dual problem of (PP)

$$\max_{w\in W} -g^*(-L^*w) - f^*(y) \qquad \text{(DP)}$$

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- Both (PP) and (DP) are solved e.g. with the Condat-Vu algorithm[8]:

$$\begin{cases} u_{k+1} = u_k - \tau\nabla f(u_k) - \tau L^* w_k \\ w_{k+1} = \text{prox}_{\sigma g^*}(w_k + \sigma L_{u_{k+1}}) \end{cases}$$

[8] Condat. J. Optim. Theory Appl. 2013

- Total variation-regularized reconstruction:

$$f(u) = \frac{1}{2}\|\mathcal{R}u - v\|_2^2, \quad g(w) = \lambda\|w\|_{2,1}, \quad L = \nabla,$$

$$\nabla f(u) = \mathcal{R}^*(\mathcal{R}u - v), \quad L^* = -\mathrm{div}, \quad \mathrm{prox}_{\sigma g^*}(w) = \Pi_{\|w\|_{2,\infty} \leq \lambda}$$

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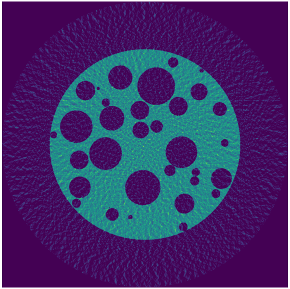
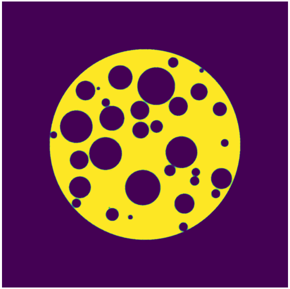
- Ex: 60 noiseless projections



FBP



Condat-Vu

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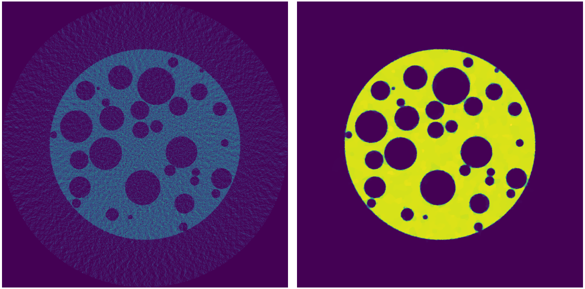
- Ex: 60 projections with Poison noisse



FBP Condat-Vu

- Goal: Fast, accurate and automatic tomography-based in-line industrial inspection
- Convergence analysis of different algorithms to design a new 'tomography-adapted' algorithm
- Resolution analysis needs to be done with different f, g, L
- $\mathcal{R}$ and $\mathcal{R}^*$ are expensive, we need faster approximations

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- Convergence analysis of different algorithms to design a new 'tomography-adapted' algorithm
- Resolution analysis needs to be done with different f, g, L
- $\mathcal{R}$ and $\mathcal{R}^*$ are expensive, we need faster approximations
- If challenging data, combine different regularizers by solving

$$\min_{u \in U} f(u) + g(Lu) + h(Ku), \quad f, g, h \text{ convex}, \quad L, K \text{ linear}$$

$\rightarrow$ gracias !